

Sequences & Series (1-4)

BONUS Review

Section 1

⑥ $S_5 = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} =$

(a)

$$S_{10} = \sum_{n=0}^9 \frac{1}{n!} =$$

[2.708]

$$\sum_{n=0}^{\infty} \frac{1}{n!} \Rightarrow \text{converges to } ⑦$$

⑥ $3 - 1 + \frac{1}{3} - \frac{1}{9} + \dots = 3(-\frac{1}{3})^n$

Geometric series. $a=3$ & $r=-\frac{1}{3}$

$$S_5 = 3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} = \left(\frac{61}{27}\right) \approx 2.259$$

$$S_{10} = \sum_{n=0}^9 3(-\frac{1}{3})^n \approx \left(\frac{14,762}{6,561}\right) \approx 2.24996$$

$$S_{\infty} = \frac{3}{1+\frac{1}{3}} = \frac{3}{\frac{4}{3}} = \left(\frac{9}{4}\right)$$

⑦ $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \Rightarrow \text{converges to } \approx .368$

$$S_5 = \frac{1}{0!} + \frac{-1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} = \left(\frac{3}{8}\right) = .375$$

$$S_{10} = \sum_{n=0}^9 \frac{(-1)^n}{n!} = .368$$

⑧ $2 - 4 + 8 - 16 + 32 - \dots$

$$a_n = (2)^n \cdot (-1)^{n+1} \text{ with } n \text{ starting at 1}$$

$\lim_{n \rightarrow \infty} a_n = \text{DNE}$ because the sequence oscillates between $+\infty$ & $-\infty$.

* Therefore, the SERIES ~~converges~~ diverges by the nth term test.

(12) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n \Rightarrow \text{This is a geometric series!}$

$$a=1 \quad r=\frac{e}{\pi}$$

* Since $r=\frac{e}{\pi} < 1$, the SERIES converges by the geometric series test.

(10) $\sum_{n=1}^{\infty} \cos(\frac{1}{n})$ * By the n th term test

Since $\lim_{n \rightarrow \infty} \cos(\frac{1}{n}) = 1 \neq 0$

(12) The series $\sum_{n=1}^{\infty} \cos(\frac{1}{n})$ DIVERGES

See previous page

(14) $\sum_{n=1}^{\infty} \sin(n)$

* By the n th term test,
Since $\lim_{n \rightarrow \infty} \sin(n) = \text{DNE}$

The $\sum_{n=1}^{\infty} \sin(n)$ DIVERGES

(18) $\sum_{n=0}^{\infty} 2\left(\frac{1}{8}\right)^n$ Converges
Since $r = \frac{1}{8} < 1$

$$S_{\infty} = \frac{2}{1-\frac{1}{8}} = \frac{2}{7/8} = \boxed{16/7}$$

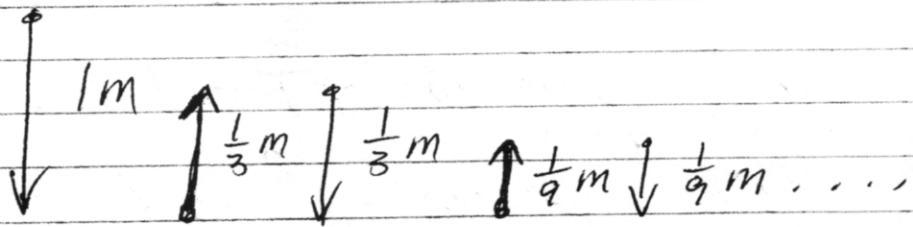
(20) $\sum_{n=0}^{\infty} \frac{4^{n+1}}{5^n} = \sum_{n=0}^{\infty} \frac{4^n \cdot 4}{5^n} =$

$$\sum_{n=0}^{\infty} 4\left(\frac{4}{5}\right)^n \text{ so it Converges}$$

since $r = 4/5 < 1$

$$S_{\infty} = \frac{4}{1-4/5} = \frac{4}{1/5} = \boxed{20}$$

(28) Initial height = 1m



$$\text{Up} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} \dots$$

$$\text{Down} = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} \dots$$

$$\text{Up} = \sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{1}{3}\right)^n \quad a = \frac{1}{3} \\ r = \frac{1}{3}$$

$$S_{\infty} = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{3} \cdot \frac{3}{2} = \boxed{\frac{1}{2}}$$

$$\text{Down} = \sum_{n=0}^{\infty} 1 \left(\frac{1}{3}\right)^n \quad a = 1 \\ r = \frac{1}{3}$$

$$S_{\infty} = \frac{1}{1 - \frac{1}{3}} = \frac{1}{\frac{2}{3}} = \boxed{\frac{3}{2}}$$

$$\text{Up} + \text{Down} = \frac{1}{2} + \frac{3}{2} = \boxed{2m}$$

(22) $\sum_{n=10}^{\infty} \left(\frac{3}{4}\right)^n$ Converges since $\left(\frac{3}{4}\right) = r < 1$

$$S_{\infty} = \frac{\left(\frac{3}{4}\right)^{10}}{1-\frac{3}{4}} = \frac{3^{10}}{4^{10}} \cdot \frac{4}{1} = \boxed{\frac{3^{10}}{4^9} \approx .225}$$

(26) $0.\overline{9} = 0.9 + 0.09 + 0.009 + 0.0009 + \dots$

$$0.\overline{9} = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{10,000} \dots$$

$$a = 9/10 \quad r = 1/10$$

(28) *see previous page* $\sum_{n=0}^{\infty} \frac{9}{10} \left(\frac{1}{10}\right)^n = \frac{9/10}{1-1/10} = \frac{9/10}{9/10} = 1$

(30) $1 + 7 + 49 + 343 + 2401$
man wives sacks cats kits

$$1 + 1(7)^1 + 1(7)^2 + 1(7)^3 + 1(7)^4$$

$$\sum_{n=0}^4 1(7)^n = \boxed{2801}$$

Section 2

(4) $\frac{1}{1+x} = 1 + 1(-x)^1 + 1(-x)^2 + 1(-x)^3 + \dots + 1(-x)^n$

$$a = 1 \quad r = -x$$

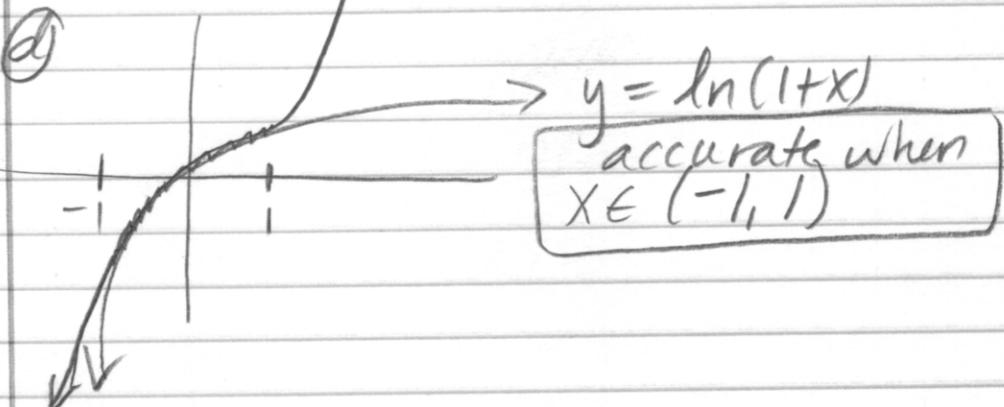
(a) $\int \frac{1}{1+x} dx = \int 1 - x + x^2 - x^3 + x^4 - x^5 + (-1)^n (x)^n dx$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$$

(b) $\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5$

$$\ln(1-(-x)) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$$

① yes! $P_5(x)$



$$\textcircled{2} \quad \ln(0.8) = \ln(1 + -0.2) \approx P_5(-0.2)$$

$$P_5(-0.2) = -0.223 \quad \ln(0.8) = -0.223$$

$$\ln(1.8) = \ln(1 + 0.8) \approx P_5(0.8)$$

$$P_5(0.8) = 0.614 \quad \ln(1.8) = 0.588$$

$$\ln(5) = \ln(1 + 4) \approx P_5(4)$$

$$P_5(4) = 158.1\bar{3} \quad \ln(5) = 1.609$$

- * $\ln(0.8)$ is closest because $x = -0.2$ is closest to the center ($a = 0$)
- * $\ln(5)$ has the worst estimate because $x = 4$ is furthest from the center ($a = 0$)

Section 3

$$\textcircled{2} \quad \textcircled{b} \quad f(x) = e^{x^2} \quad \textcircled{d} \quad \text{using this...}$$

$$e^{x^2} \approx 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \dots + \frac{x^n}{n!}$$

$$e^{x^2} \approx 1 + x^2 + \frac{x^4}{2!}$$

* Just put x^2 in for all $x^5 \rightarrow \text{NICE!}$

OR

$$f'(x) = 2x \cdot e^{x^2}$$

$$f''(x) = 2e^{x^2} + 2x \cdot 2xe^{x^2}$$

$$f'''(x) = 2e^{x^2}(1 + 2x^2)$$

$$f''''(x) = 4xe^{x^2}(1 + 2x^2) + 4x(2e^{x^2})$$

②(b) continued

$$f'''(x) = 4xe^{x^2}(1+2x^2+2)$$

$$f'''(x) = 4xe^{x^2}(2x^2+3) = e^{x^2}(8x^3+12x)$$

$$f^4(x) = 2xe^{x^2}(8x^3+12x) + (24x^2+12)e^{x^2}$$

$$f^4(x) = 2e^{x^2}(8x^4+12x^2+12x^2+6)$$

$$f^4(x) = 2e^{x^2}(8x^4+24x^2+6)$$

$$f^5(x) = 4xe^{x^2}(8x^4+24x^2+6) + (32x^3+48x) \cdot 2e^{x^2}$$

$$f(0) = 1 \quad f''(0) = 2(1)(1) = 2 \quad f^4(0) = 2(6) = 12$$

$$f'(0) = 0 \quad f'''(0) = 1(0) = 0 \quad f^5(0) = 0$$

$$P_5(x) = \frac{0}{5!}(x)^5 + \frac{12}{4!}x^4 + \frac{0}{3!}x^3 + \frac{2}{2!}x^2 + \frac{0}{1!}x + 1$$

$$\underbrace{P_5(x) = \frac{1}{2}x^4 + x^2 + 1}_{\text{5th order}} \rightarrow \text{Notice this}$$

is only 4th degree
even though
we used 5th derivative.

$$\textcircled{2} \textcircled{c} \quad f(x) = x^{1/3} \quad a = -1$$

$$f'(x) = \frac{1}{3}x^{-2/3} \quad f(-1) = -1$$

$$f''(x) = -\frac{2}{9}x^{-5/3} \quad f'(-1) = \frac{1}{3(-1)^2} = \frac{1}{3}$$

$$f''(-1) = \frac{-2}{9(-1)^5} = \frac{2}{9}$$

$$P_2(x) = \frac{2}{9} \cdot \frac{1}{2!}(x+1)^2 + \frac{1}{3}(x+1)' - 1$$

$$\underbrace{P_2(x) = \frac{1}{9}(x+1)^2 + \frac{1}{3}(x+1) - 1}_{\text{P}_2(x)}$$

$$\textcircled{2} \textcircled{d} \quad f(x) = x^2 + 7x - 4 \quad a = 1$$

$$f'(x) = 2x + 7 \quad f(1) = 4$$

$$f''(x) = 2 \quad f'(1) = 9$$

$$\boxed{P_2(x) = \frac{2}{2!}(x-1)^2 + 9(x-1) + 4} \quad f''(1) = 2$$

$$\textcircled{10} \quad f(0) = 3, \quad f'(0) = -8, \quad f''(0) = 5 \quad f'''(0) = 2$$

$$P_2(x) = \frac{5}{2!}x^2 + \frac{-8}{1!}(x)' + 3$$

$$P_2(x) = \frac{5}{2}x^2 - 8x + 3$$

$$P_3(x) = \frac{2}{3!}x^3 + \frac{5}{2!}x^2 - 8x + 3$$

$$P_2(0.3) = 0.825$$

$$P_3(0.3) = 0.834$$

* $P_3(0.3)$ should be better estimate of $f(0.3)$

Section 4

$$\textcircled{2} \quad f(x) = \ln(1+x) \quad (\text{see } \#4 \text{ in Section 2})$$

$$* \quad P_5(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$$

OR $f(0) = 0 \quad P_5(x) = \frac{24}{5!}x^5 - \frac{6}{4!}x^4 + \frac{2}{3!}x^3 - \frac{1}{2}x^2 + x$

$$f'(x) = \frac{1}{1+x} \quad f'(0) = 1$$

$$P_5(x) = \frac{1}{5}x^5 - \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{2}x^2 + x$$

$$f''(x) = -\frac{1}{(1+x)^2} \quad f''(0) = -1$$

$$\ln(1.2) = \ln(1+0.2)$$

$$f'''(0) = 2$$

$$\text{So... } x = 0.2$$

$$f''''(x) = \frac{2}{(1+x)^3}$$

$$f''''(0) = -6$$

$$P_5(0.2) = 0.182$$

$$f''''(x) = \frac{-6}{(1+x)^4} \quad f''''(0) = 24$$

$$|R_5(0.2)| \leq \frac{120}{6!}(0.2)^6$$

$$f''''(x) = \frac{24}{(1+x)^5} \quad f''''(0) = -120$$

$$|\text{Error}| \leq 1.06 \times 10^{-5}$$

$$(f''''(x) = \frac{-120}{(1+x)^6}) \rightarrow |f''''(0)| = 120 \quad \text{so } M = 120$$

$$④ f(x) = \sin x \quad a=0 \quad x=-0.3$$

$$P_3(x) = x - \frac{x^3}{3!}$$

$$P_3(-0.3) = -0.296 \approx \sin(-0.3)$$

$$|R_3(-0.3)| \leq \frac{1}{4!} (-0.3)^4$$

$$|\text{Error}| \leq 3.375 \times 10^{-4}$$

or

$$-3.375 \times 10^{-4} \leq \text{error} \leq 3.375 \times 10^{-4}$$

$$\begin{array}{ll} ⑥ & f(x) = \sin x \quad x=1 \quad a=\pi/3 \\ & f'(x) = \cos x \quad f(\pi/3) = \sqrt{3}/2 \\ & f''(x) = -\sin x \quad f'(\pi/3) = 1/2 \\ & & f''(\pi/3) = -\sqrt{3}/2 \end{array}$$

$$P_2(x) = -\frac{\sqrt{3}}{2} \cdot \frac{1}{2!} (x - \pi/3)^2 + \frac{1}{2} (x - \pi/3) + \frac{\sqrt{3}}{2}$$

ⓐ $P_2(1) = 0.841 \approx f(1)$

$$\textcircled{b} \quad |R_n(1)| \leq \underbrace{\frac{1}{(n+1)!} (1 - \pi/3)^{(n+1)}}_{\text{Put in } y_1 \text{ on calc \& use table to plug in "n".}} \leq 10^{-9}$$

$n=5^{\text{th}}$
degree